**I.**Compute the first-order partial derivatives*.*

**1.** *z* = *x*2 + *y*2 **2.** *z* = *x*4*y*3 **3.** *z* = *x*4*y* + *xy*−2 **4.** *V* = *πr*2*h*

**5.** *z* = **6.** *z* = **7.** *S* = tan−1*(wz)* **8.** *Q* = *reθ*

**9.** *z* = ln(*x*2 + *y*2)**10.** *z* = *yx*

**II.** Show that the following functions satisfy the **Laplace equation** *uxx* + *uyy* = 0:

**(a)** *u(x, y)* = *ex* cos *y* **(b)** *u(x, y)* = tan−1 *yx* **(c)** *u(x, y)* = ln(*x*2 + *y*2)

**III.** Find all constants *a, b* such that *u(x, y)* = cos(*ax*) *eby* satisfies the Laplace equation

*uxx* + *uyy* = 0.

**IV.** Show that the following functions satisfy the **heat equation** *ut* = *uxx* for *t >* 0 and all *x*:

(a) *z* = *e*−*t* sin(*x/c*)(b) *z* = *e*−*t* cos(*x/c*)(c) *z* = π %FontSize=12
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\[
e^{-x^2/(4t)}
\]
\end{document}**.**

**V.** (a) Show that a function of the form *u*(*x, t*)= *f*(*x* + *ct*) satisfies the wave equation.

(b) Show that a function of the form *u*(*x, t*)= *g*(*x* − *ct*) satisfies the wave equation.

(c) Show that a function of the form *u*(*x, t*)= *f*(*x* + *ct*)+ *g*(*x* − *ct*)

satisfies the wave equation.

VI. Find the linear, *L*(*x, y*). Compare the value of the approximation *L*(0*.*9*,* 0*.*2) with the exact value of the function *f* (0*.*9*,* 0*.*2).

**a.** *f* (*x, y*) = **b.** *F* (*x, y*) = *x*2*y* **c.** *f* (*x, y*) = *xe*−*y*

**d.** *F*(*x, y*)= *ex* sin *y* + *ey* sin *x* **e.** *f* (*x, y*) = sin(*x* − 1) cos *y*

VII. Compute the differential *dz* or *dw* of the function. ■

**1.** *z* = 7*x* − 2*y*  **2.** *z* = *exy*

**3.** *z* = *x*3*y*2 **4.** *z* = tan−1 *xy*

**5.** *z* = *e*−3*x* cos 6*y* **6.** *w* = 8*x* − 3*y* + 4*z*

**7.** *w* = *exyz* **8.** *w* = *x*3*y*2*z*

**VIII.** Use the Chain Rule to calculate *f (***c**(*t*))*.*

**1.** *f* (*x, y*)= 3*x* − 7*y*, **c**(*t*)= *(*cos *t,* sin *t)*, *t* = 0

**2.** *f* (*x, y*) = 3*x* − 7*y*, **c***(t)* = *(t*2*, t*3*)*, *t* = 2

**3.** *f* (*x, y*) = *x*2 − 3*xy*, **c***(t)* = *(*cos *t,* sin *t)*, *t* = 0

**4.** *f* (*x, y*) = *x*2 − 3*xy*, **c***(t)* = *(*cos *t,* sin *t)*, *t* =

**5.** *f* (*x, y*) = sin*(xy)*, **c***(t)* = *(e*2*t, e*3*t )*, *t* = 0

**6.** *f* (*x, y*) = cos*(y* − *x)*, **c***(t)* = *(et, e*2*t )*, *t* = ln 3

**7.** *f (x, y)* = *xey* , **c**(*t*)= *(t*2*, t*2 − 4*t)*, *t* = 0

**8.** *f* (*x, y*) = ln *x* + ln *y*, **c**(*t*)= *(*cos *t, t*2*)*, *t* =

**IX.** Find an equation of the tangent plane at the given point.

**1.** *f (x, y)* = *x*2*y* + *xy*3, (2, 1)

**2.** *f (x, y)* = , (4, 4)

**3.** *f (x, y)* = *x*2 + *y*−2, (4, 1)

**4.** *G*(*u,w*)= sin(*uw*), (π/6*,* 1)

**5.** *g(x, y)* = *ex/y*, (2, 1)

**6.** *f (x, y)* = ln*(*4*x*2 − *y*2*)*, (1, 1)

**7.** Find the linearization *L(x, y)* of *f (x, y)* = *x*2*y*3 at (*a*, *b*)= (2, 1). Use it to estimate *f* (2.01, 1.02)and *f* (1.97, 1.01)and compare with actual values.